Example Problem: Spherical tank

Plane 1: top of sphere

Plane 2: bottom of sphere

Plane 3: exit point at pipe bottom

h(t): height of liquid in tank at any instant t

Volume of the differential disk = πr2dh

r = r(t)

Y = 2R - h

X = R-y = R - (2R - h) = h - R

r2 = R2 - x2 = R2 - (h - R)2 = 2Rh - h2

dV = π(2Rh - h2)dh

dM = ρdV = ρπ(2Rh - h2)dh

Mtotal = ∫0hρπ(2Rh - h2)dh

Since the mass change in the tank is flowing out of pipe:

Rate of change of mass in tank = mass flow rate out of pipe

Hagen Poiseuille Equation

ΔP = 128μLΦ/πD4

P2 - P3 = 128μLΦ/πD4

Δw = w1 - w2

Δw = -w2

W3 = mass flow rate exiting the pipe = Φρ = (P2-P3)πD4ρ/128μL

-w2 = w3 = (P2-P3)πD4ρ/128μL

πρR(2h - h2/R) dh/dt = w2

P3 = ρg(h+L) + P0

P2 = P0

w3 = ρg(h+L)πD4ρ/128μL

πρR(2h - h2/R) dh/d = ρg(h+L)πD4ρ/128μL

At t = 0, h = 2R

At t = tefflux, h = 0

Macroscopic Mechanical Energy Balance

Unsteady State Macroscopic Mechanical Energy Balance (Engineering Bernoulli Equation)

d/dt (KEtotal + PEtotal) = (1/2ρ1v12v1s1 - 1/2ρ2v22v2s2) + (ρ1Φ1v1s1 - ρ2Φ2v2s2) + Wm + (p1s1<v1> - p2s2<v2>) +∫v(t)p(⛛\*v)dv + ∫v(t)(τ\*⛛v)dv

d/dt (KEtotal + PEtotal) = 1/2(ρ1<v13>s1 - ρ2<v23>s2) + (Φ1w1 - Φ2w2) + Wm + (P1w1/ρ1 - P2w2/ρ2) - Ec - Ev